

# Supercompression of Intense Ion Beams in a Collapsing Pinch

F. Winterberg

Desert Research Institute University of Nevada System, Reno, Nevada 89507,  
July 1976

(Z. Naturforsch. **32a**, 840–843 [1977]; received May 18, 1977)

Intense ion beams produced in magnetically insulated diodes become fast plasma jets in the drift tube. If a pinch discharge is made over these fast jets they can be magnetically compressed to very small diameters of less than  $10^{-2}$  cm. Such fast highly focused plasma beams may have important applications for the ignition of thermonuclear microexplosions.

Methods for the generation of intense megavolt ion beams by a modification of the existing technique for the generation of intense relativistic electron beams have been proposed, and are 1) magnetic diode insulation<sup>1</sup>, 2) the reflex triode<sup>2,3</sup> and 3) magnetic self-insulation<sup>4</sup>. With these methods ion currents up to  $10^5$  A are well within reach with the future prospect of megampere currents. In all of these methods the ion beam emerges from the diode as a space charge neutralized high speed flow of plasma, in which the ions, due to electrostatic forces, have picked up and are dragging along with them electrons\*. Because of the space charge limitation in the diode the beams have large diameters. Therefore, to apply these intense pulsed beams for the ignition of thermonuclear microexplosions a method to focus them down to diameters 1 mm or less is of crucial importance. We will here propose a method which promises the focusing of these intense beams down to diameters of  $\sim 10^{-3}$  cm. The idea is explained in Figure 1. After closing the switch  $S_1$  connecting the high voltage Marx generator storage capacitors  $C_1$  to the diode D, an ion beam of total current  $I$  is accelerated from the anode A to the cathode C and then injected as a space charge neutralized plasma beam into the drift tube T which is filled by a tenuous background gas. Immediately after the front of the ion beam enters the drift tube T, the switch  $S_2$  is closed axially discharging the large capacitor bank  $C_2$  over the background gas of the drift tube and the plasma beam entering the drift tube. The discharge results in a large current  $I'$  from C to C' pinching the plasma beam as it moves to the right. If the pinch current is above the critical Pease-Braginskii current, at which the bremsstrahlungs-losses ex-

ceed the Joule heating rate, the pinch will rapidly shrink and radially compress the beam. For a hydrogen plasma this critical current is  $\sim 1.5 \times 10^6$  A, but it can be substantially smaller for a high-Z plasma or a hydrogen plasma with high-Z impurities. If the pinch current is above this critical value the pinch will shrink until it becomes optically opaque. From there on further shrinkage will be slowed down until a new equilibrium is reached at which the pinch and with it the beam assumes its smallest diameter. The radiation loss time can be made smaller than the time for pinch instabilities to develop and the beam will be therefore compressed prior to its disruption by instabilities. After the compression to its minimum radius the beam can be transported with little divergence over rather large distances.

The maximum ion current density  $j$  follows from the space charge equation ( $q$ ,  $M$  ion charge and mass,  $V$  diode voltage,  $d$  diode gap in cgs units)

$$j = (2q/M)^{1/2} V^{3/2} / 9\pi d^2 [\text{esu}] . \quad (1)$$

If the ions are accelerated to the velocity  $v_0$ , then  $(M/2)v_0^2 = qV$ . The initial ion number density of the beam is  $n_0 = j/qv_0$  or according to Eq. (1)

$$n_0 = V/9\pi q d^2 . \quad (2)$$

Take the following example:  $V = 3 \times 10^6$  Volt =  $10^4$  esu,  $d = 0.1$  cm,  $q = e$  (singly ionized ions), then  $n_0 \cong 10^{14}$  cm<sup>-3</sup>. For an ion mass  $M = A M_H \cong 200 M_H$  ( $M_H$  hydrogen mass), one has  $v_0 \cong 10^8$  cm/sec and  $j \cong 2 \times 10^3$  A/cm<sup>2</sup> and an emitter area of  $5 \times 10^3$  cm<sup>2</sup>, corresponding to an initial beam radius  $r_0 \cong 40$  cm, would give the ion beam current  $I \cong 10^7$  A. For a beam pulse of  $\tau \cong 10^{-8}$  sec, the beam energy and beam power would be  $3 \times 10^5$  J and  $3 \times 10^{13}$  Watt.

For the pinch of current  $I'$  it is assumed that during its radial contraction the inertial term in the radial component of the equation of motion can be

\* These space charge neutralized beams are thus in fact fast plasma jets and derive their customary name only from their origin in a high voltage diode.



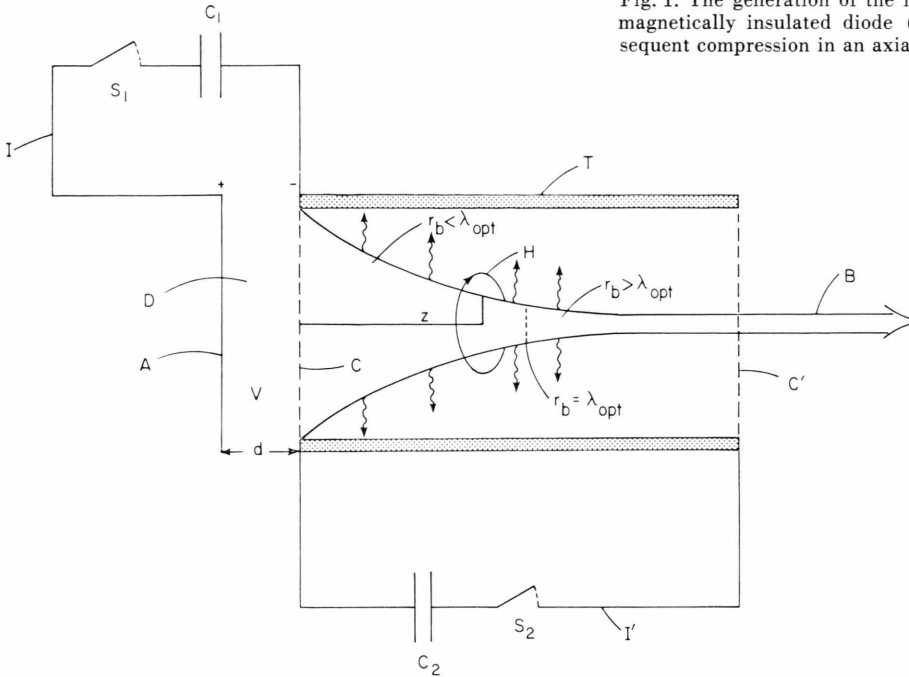


Fig. 1. The generation of the intense ion beam in the pulsed magnetically insulated diode (or reflex triode) and its subsequent compression in an axial pinch discharge.

neglected. This leads to the Bennett relation

$$I'^2 = 4 c^2 N k T [\text{esu}] , \quad (3)$$

where  $N = \pi r_b^2 n$  are the number of ions per unit beam length with  $r_b$  the beam radius.

The pinched beam will shrink as long as the radiative energy losses  $P_r$  exceed the Joule heat rate  $P_Q$ . Assuming a constant current distribution over the beam the Joule heat rate per unit beam length is given by

$$P_Q = I'^2 / \pi r_b^2 \sigma , \quad (4)$$

where  $\sigma = a T^{3/2} [\text{s}^{-1}]$  ( $a \cong 0.9 \times 10^7 [\text{cgs}]$ ) is the electrical conductivity. The radiative energy loss has to be computed for the two limiting cases of a transparent and opaque beam. If we assume (in conformity with the Bennett relation) a beam temperature of  $T \sim 10^5 \text{ }^\circ\text{K}$ , we can use for the optical cross section the rough estimate  $\sigma_{\text{opt}} \sim 10^{-18} \text{ cm}^2$ . The beam is transparent for  $\lambda_{\text{opt}} = 1/n \sigma_{\text{opt}} > r_b$  otherwise opaque. For the given example of the initial beam parameters  $n_0 \cong 10^{14} \text{ cm}^{-3}$ , it is  $\lambda_{\text{opt}} \cong 10^4 \text{ cm}$ , hence  $\lambda_{\text{opt}} > r_b = r_0 = 40 \text{ cm}$ , and the beam transparent. It will be shown that the beam velocity is practically unchanged during the beam compression implying that  $n = n_0 (r_0/r_b)^2$ . Therefore,  $\lambda_{\text{opt}} \sim r_b$  at  $r_b \sim n_0 r_0^2 \sigma_{\text{opt}} \sim 0.1 \text{ cm}$ , and the beam is opaque for

$r_b \lesssim 0.1 \text{ cm}$ . For the transparent beam the radiation losses  $P_r^{(1)}$  are given by

$$P_r^{(1)} = f b n^2 T^{1/2} \pi r_b^2 [\text{erg/cm sec}] , \quad (5)$$

where  $b = 1.42 \times 10^{-27} [\text{cgs}]$  and where  $f$  is a factor which for a pure hydrogen plasma is  $f = 1$ . For a non-hydrogenic plasma<sup>5</sup>, for example with  $Z = 10$ ,  $T = 10^6 \text{ }^\circ\text{K}$  one has  $f \cong 5 \times 10^4$  and for  $Z = 10$ ,  $T = 10^5 \text{ }^\circ\text{K}$  one has  $f = 5 \times 10^6$ . Larger  $Z$ -values increase the value of  $f$ . Large  $f$ -values can be also reached by adding high- $Z$  impurities to a hydrogen beam and which are sufficient for a strong radiative cooling as required here<sup>\*\*</sup>. In case of an opaque beam the radiation losses  $P_r^{(2)}$  are given by Stefan-Boltzmann's law, hence

$$P_r^{(2)} = 2 \pi r_b \kappa T^4 [\text{erg/cm sec}] , \quad (6)$$

where  $\kappa = 5.75 \times 10^{-5} [\text{erg/cm}^2 \text{ sec } ^\circ\text{K}^4]$ .

From the inequality  $P_r^{(1)} > P_Q$ , which is necessary for beam collapse, one has [by using  $n = N/\pi r_b^2$  and the Bennett relation (3)]

$$I' > 4 c^2 k / \sqrt{a b f} = I_{\text{PB}} / \sqrt{f} , \quad (7)$$

where  $I_{\text{PB}} \cong 1.5 \times 10^6 \text{ A}$  is the Pease-Braginskii current for a hydrogen plasma. With  $f = 5 \times 10^4$ , a cur-

<sup>\*\*</sup> This  $Z$ -value is here to be understood the atomic number and not the degree of ionization which under the made assumption remains to be single.

rent  $I' > 10^4$  A would thus be sufficient to compress the beam.

After the beam has become opaque further shrinkage stops if  $P_r^{(2)} = P_\Omega$ , or

$$I'^2/\pi r_b^2 \sigma = 2 \pi r_b \propto T^4. \quad (8)$$

Using the Bennett relation the temperature can be eliminated from Eq. (8) with the result

$$r_b = r_{\min} = \frac{4^{5/3}}{\pi^{2/3}} \frac{c^{11/3} k^{11/6} N^{11/6}}{a^{1/3} \kappa^{1/3} I'^3}. \quad (9)$$

Expressing  $I'$  in Ampere one obtains

$$r_{\min} \cong 5 \times 10^{-20} N^{11/6} / I'^3 \text{ [cm]}, \quad (10)$$

and for the beam temperature

$$T \cong 1.8 \times 10^{13} I'^2 / N \text{ [}^\circ\text{K]}. \quad (11)$$

Finally, we have to show that the beam forward velocity is little affected by the beam compression. The Bennett relation expresses isothermal beam flow and using the nozzle flow approximation neglecting radial beam accelerations, the axial equation of motion is

$$\frac{1}{2} \frac{dv^2}{dz} = - \frac{1}{\varrho} \frac{dp}{dz} = - \frac{2kT}{AM_H} \frac{d \ln p}{dz}, \quad (12)$$

with the solution

$$v = v_0 \left[ 1 - \frac{4}{3} \left( \frac{v_{\text{th}}}{v_0} \right)^2 \ln \left( \frac{p}{p_0} \right) \right]^{1/2}, \quad (13)$$

where  $v_{\text{th}} = (3kT/AM_H)^{1/2}$  is the thermal beam velocity,  $p_0$  the initial and  $p$  the final beam pressure. Since  $p = H^2/8\pi$  and  $H = 2I'/rc$  it follows that  $p/p_0 = (r_0/r_b)^2$ . For  $T \sim 10^5$  °K,  $v_{\text{th}} \sim 10^6$  cm/sec and  $(v_0/v_{\text{th}})^2 \sim 10^{-4}$ . This means that even if  $r_b \sim 10^{-4}$  and  $r_0/r_b \sim 10^5$ ,  $v \cong v_0(1 - 2 \times 10^{-3})$  and which shows that the beam forward velocity remains practically constant. Because of the constancy of  $v$ , then also  $N = \pi r_0^2 n_0 \cong 5 \times 10^{17} \text{ cm}^{-1}$  remains constant. With this value of  $N$  and putting for example  $I' = 10^5$  A one has from Eqs. (10) and (11),  $r_{\min} \cong 10^{-2}$  cm,  $T \cong 4 \times 10^5$  °K. This rather low temperature makes the plasma highly collisional and makes it unlikely that the critical current for pinch collapse will be increased due to anomalous resistivity. The radiative power according to Eq. (6), and which has to be supplied by the bank  $C_2$ , is  $P_r^{(2)} \sim 10^{17} \text{ erg/cm sec}$ . For a beam pulse of  $\tau \sim 10^{-8}$  sec this is  $10^9 \text{ erg/cm}$  and for a beam velocity of  $10^8 \text{ cm/sec}$  the beam length is  $\sim 1$  cm such that the total radiatively lost energy is  $\sim 100$  J. By going to  $I' =$

$2 \times 10^5$  A one has  $r_{\min} \cong 10^{-3}$  cm,  $T \cong 1.6 \times 10^6$  °K, and for the radiative energy loss  $\sim 2 \times 10^4$  J.

The growth time for the pinch instability is given by

$$\tau_i \cong 20 r_b / v_{\text{th}}. \quad (14)$$

For the pinch to be unaffected by instability during collapse this time has to be larger than the radiative loss time for bremsstrahlung

$$\begin{aligned} \tau_R &= \frac{3 n k T}{f b n^2 T^{1/2}} = \frac{3 \pi k T}{b} \frac{T^{1/2} r_b^2}{f N} \\ &\cong 10^{12} T^{1/2} r_b^2 / f N \text{ [sec]}. \end{aligned} \quad (15)$$

We compare these times at  $r_b \sim 0.1$  where the beam becomes opaque and its collapse being slowed down. Putting  $T \cong 4 \times 10^5$  °K,  $N = 5 \times 10^{17} \text{ cm}^{-1}$ ,  $v_{\text{th}} \sim 10^6 \text{ cm/sec}$  and  $f = 10^4$  one has  $\tau_i \sim 10^{-7}$  sec and  $\tau_R \sim 2 \times 10^{-6}$  sec, such that  $\tau_i > \tau_R$  and the pinch instability should pose no serious problem. The collapse time  $\tau_c$  from the original beam radius  $r_b = r_0$  is determined by the Alfvén speed and one has  $\tau_c \sim r_0/v_A$ ,  $v_A = H/\sqrt{4\pi\varrho}$ . In order to have a fast pinch collapse  $H$  should be initially large and thus  $I'$  since  $H = 0.2 I'/r_0$ . The current will be initially anyway much higher since the resistance of the isothermal pinch column increases in proportion to  $r_b^{-2}$ . If for example, initially  $I' \sim 5 \times 10^6$  A, then with  $\varrho = n_0 A M_H = 3.3 \times 10^{-8} \text{ g/cm}^3$  one has  $v_A = 5 \times 10^7 \text{ cm/sec}$  and  $\tau_c \cong 10^{-6}$  sec. To obtain complete beam collapse the pinch column must have the length  $l \sim v_0 \tau_c \sim 10^2$  cm. The large initial current can be substantially reduced if the beam is first projected into a magnetic mirror precompressing the beam. For a precompression corresponding to a beam radius reduction by a factor 10 and which could be done by a magnetic mirror field of  $\sim 3 \times 10^4$  Gauss, the initial pinch current could be reduced to  $\sim 5 \times 10^5$  A.

Beams focused down to  $r_b \sim 0.1$  cm can be used for beam induced implosions<sup>6</sup>. If a beam with a radius of  $r_b < 10^{-2}$  cm is focused onto a solid target, the low energy beam electrons will be scattered stripping the beam of its current neutralizing electron flow. As a consequence, very large magnetic fields up to  $\sim 10^9$  Gauss can be set up around the beam. These large magnetic beam fields can trap charged fusion products along the beam channel in the same way as it had been discussed for electron beams<sup>7</sup>. Because of this entrapment of charged fusion products it is conceivable that the focused beams may be used to set off a fusion chain reaction ultimately with the  $B^{11} + H \rightarrow 3 \text{ He}^4$  nuclear reaction which is free of any neutron radiation.

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